APPLIED MATHEMATICS

QUESTION BANK

ADVANCED LEARNER

1) Examine for consistency and solve, if consistence

$$x + y + z = 3$$

$$2x - y + 3z = 1$$

$$4x + y + 5z = 2$$

$$3x - 2y + z = 4$$

2) Using De-Moivre's theorem prove that:

$$(1+i)^8 + (1-i)^8 = 32$$

3) Solve $\cos x \frac{dy}{dx} + y \sin x = \sec^2 x$

4) Solve: $(D^2 + 3D + 2)y = e^{ex}$

5) Find Laplace transform of the following:

$$\frac{dy}{dt} + 3y + 2\int_0^t y(t)dt = t; \quad given\ y(0) = 0.$$

6) Define Heaviside unit step function and hence find Laplace transform of $f(t) = (t-1)^2 H(t-1)$

7) Evaluate $\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{dx\,dy}{\sqrt{a^2-x^2-y^2}}$.

8) Find the volume of the region bounded by the surfaces $y = x^2$, $x = y^2$ and planes $z = 0, z = 3$

9) Evaluate $\int_0^{\pi} x\ sin^6 x\ dx$.

10) Show that : $\int_0^{\infty} \frac{1-e^{-ax}}{x} . e^{-x}\ dx = \log(a+1)$.

SLOW LEARNER

1) Express the matrix A as sum of symmetric and skew symmetric matrix, where

$$A = \begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}$$

2) If $5\sinh x - \cosh x = 5$ then find $\tanh x$.

3) Solve: $(e^{2x} + \sin x \cos y)\,dx + (\tan y + \cos x \sin y)dy = 0$

4) Solve: $(p - 2x)(p - y) = 0$

5) Define Laplace transformation and find $L[f(t)]$ where, $f(t) = t^n$.

6) Find inverse Laplace transform by using convolution theorem of $\frac{2}{(s^2+4)^2}$

7) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} x^2 y\ dy\,dx$

8) Find by double integration the area included between the curves $y^2 = 4ax$ and $x^2 = 4ay$.

9) Evaluate $\int_0^{1/2} x^3\sqrt{1-4x^2}\,dx$.

10) Define Error function and Prove that
   a) $\text{erf}(x) + erf_c(x) = 1$
   b) $erf_c(-x) + erf_c(x) = 2$

ASSIGNMENT QUESTIONS

1) Find two non- singular matrices P & Q such that PAQ is in normal form where,

$$A = \begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 1 & 4 & -5 \\ -1 & -5 & -5 & 7 \end{bmatrix}$$

2) Solve: $3e^x \tan y\,dx + (1 - e^x)\sec^2 y\,dy = 0$

3) State Second shifting property and find the Laplace transform of

$$F(t) = \begin{cases} e^{-4(t-3)}\sin 3(t-3) & t > 3 \\ 0 & t < 3 \end{cases}$$

4) Evaluate: $\iiint (x + y + z)\,dx\,dy\,dz$ over the tetrahedron bounded by the planes x=0, y=0, z=0 and x + y + z =1.

5) Using differentiation under integral sign, show that:

$$\int_0^{\infty} \frac{sinx}{x} = \frac{\pi}{2}$$

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